Self-Superposition Transmission: A Novel Method for Enhancing Performance of Convolutional Codes

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Outline

Background

- Convolutional codes have been widely used in communication systems, such as space, wireless and broadcast communications.
- The error performance of convolutional codes depends more on the constraint length rather than the number of information bits.
- Convolutional codes have been used to construct more powerful FEC codes, such as Turbo codes.

Constructing powerful FEC codes based on convolutional structure

- Hyperimposed convolutional codes. [Cheng, ICC, 1996]
- Stream-oriented Turbo codes. [Hall and Wilson, TIT, 2001]
- Laminated Turbo codes. [Huebner *et. al*, TIT, 2008]
- Convolutional LDPC codes. [Jiménez Felström and Zigangirov, TIT, 1999]
- Spatially-coupled LDPC codes. [Kudekar *et. al*, TIT, 2011]
- Block Markov superposition transmission codes. [Ma *et. al*, TIT, 2015]

Motivation

Key fact of convolutional codewords

- Coded bits are strongly dependent on each other if they are close in time domain and are loosely coupled otherwise.

Motivation

- Create certain relationships between loosely coupled coded bits in convolutional codewords.

Main Contribution

- We proposed self-superposition convolutional codes based on convolutional codes.
- The proposed self-superposition transmission does not change the codeword length nor the code rate.
- The proposed codes are suitable for short block length transmission.

Encoding of Self-Superposition Convolutional Codes

Self-superposition convolutional codes

- Convolutionally coded sequences are permuted and then superimposed with each other.

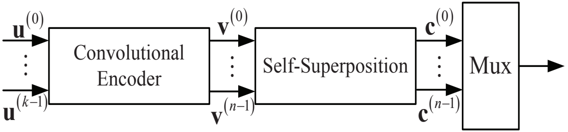

Figure: The encoder structure of self-superposition convolutional codes.

Encoding of Self-Superposition Convolutional Codes

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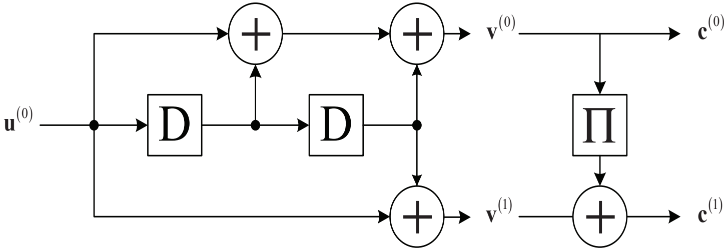


Figure: An example of the encoding of self-superposition convolutional codes based on the (7, 5) convolutional code.

Self-Superposition Transmission: Matrix Representation

$$\mathbf{G}_{\mathrm{SS}} = \begin{bmatrix} \mathbf{I} & \mathbf{\Pi}_0 & \mathbf{\Pi}_1 \\ \mathbf{0} & \mathbf{I} & \mathbf{\Pi}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{c}^{(0)} \\ \mathbf{c}^{(1)} \\ \mathbf{c}^{(2)} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \mathbf{v}^{(0)} \\ \mathbf{v}^{(1)} \\ \mathbf{v}^{(2)} \end{bmatrix}^{\mathrm{T}} \mathbf{G}_{\mathrm{SS}}$$

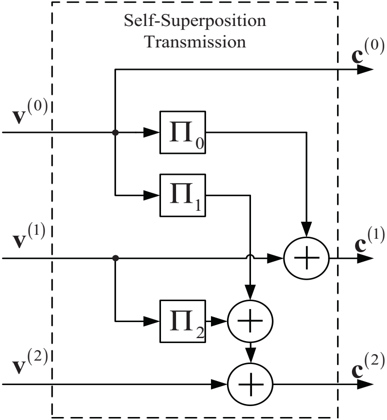


Figure: The generator matrix representation of the corresponding self-superposition transmission.

Decoding of self-superposition convolutional codes

Iterative decoder for self-superposition convolutional codes with low-complexity

- Iteration between the decoder of the self-superposition operation and the decoder of the original convolutional code.

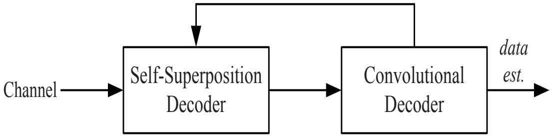


Figure: A diagram of the iterative decoding between the self-superposition transmission and the convolutional code.

Factor graph of self-superposition convolutional codes

Constraint nodes in the factor graph

- Node $\boxed{\mathrm{C}}$ denotes the original convolutional code $\mathcal{C}[n, k, v]$.
- Node $\boxed{=}$ denotes the constraint that all connecting variables must have the same value. (Similar to the variable node in a binary LDPC code.)
- Node $\boxed{+}$ denotes the constraint that the sum of all connecting variables over $\mathbb{F}_2$ must be zero. (Similar to the check node in a binary LDPC code.)
- Node $\boxed{\Pi_i}$ denotes the ith interleaver.

Iterative Decoder for Self-Superposition Convolutional Codes with Low-Complexity: Factor Graph Representation

$$\mathbf{G}_{SS} = \begin{bmatrix} \mathbf{I} & \mathbf{\Pi}_0 & \mathbf{\Pi}_1 \\ \mathbf{0} & \mathbf{I} & \mathbf{\Pi}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$

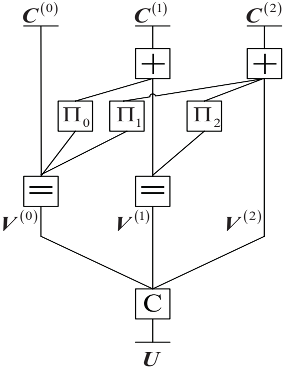
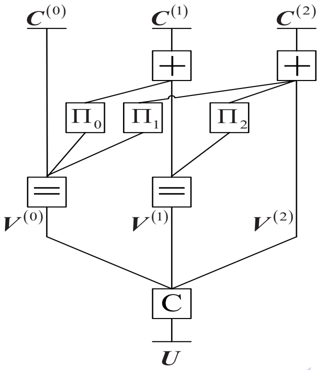


Figure: An example of a factor graph of an SSCC with the generator matrix.

Iterative Decoding Based on the Factor Graph

Decoding Scheduling:

- Forward recursion: $\boxed{+} \rightarrow \boxed{\Pi} \rightarrow \boxed{=} \rightarrow \boxed{\mathrm{C}}$.
- Backward recursion: $\boxed{\mathrm{C}} \rightarrow \boxed{=} \rightarrow \boxed{\Pi} \rightarrow \boxed{+}$.

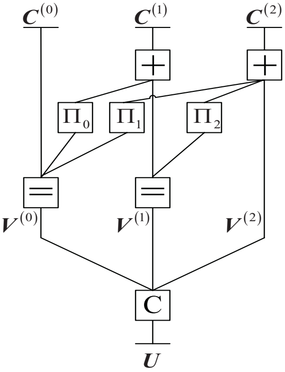


Code Construction Criteria

Successful decoding criteria for self-superposition convolutional codes

- In order to have a good iteration start, it is required that the generator matrix $\mathbf{G}_{\mathrm{SS}}$ can be arranged to be a triangular matrix by only switching its rows or columns.

$$\mathbf{G}_{\mathrm{SS}} = \begin{bmatrix} \mathbf{I} & \mathbf{\Pi}_0 & \mathbf{\Pi}_1 \\ \mathbf{0} & \mathbf{I} & \mathbf{\Pi}_2 \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}$$



Code Construction Criteria

How to choose a suitable codeword length?

- In order to have a good interleaving gain, the interleaver size should be larger than 5-7 times of the constraint length, so that the loosely coupled coded bits existed in the original convolutional codeword.

Numerical Results

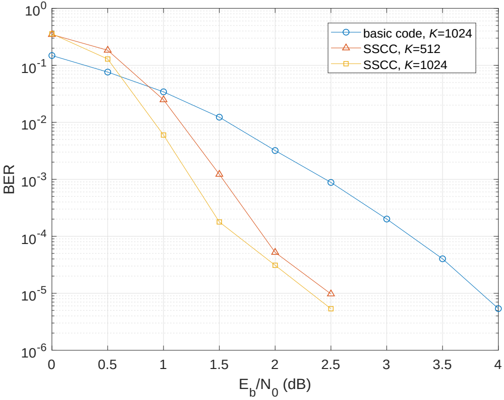


Figure: BER results of self-superposition convolutional codes with different codeword lengths and the corresponding convolutional code $\mathcal{C}[2,1,8]$, where the generator matrix is given as $\begin{bmatrix} \mathbf{I} & \mathbf{\Pi} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$.

Numerical Results

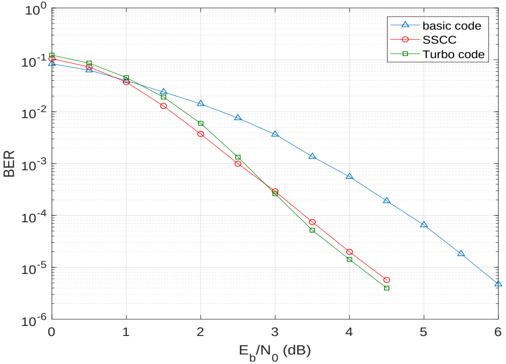


Figure: BER results of the self-superposition convolutional codes, LTE Turbo code and the corresponding convolutional code $\mathcal{C}\,[2,1,4]$ with 5 decoding iterations, where the generator matrix is $\begin{bmatrix} \mathbf{I} & \mathbf{\Pi} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$ and the length of the self-superposition convolutional codeword and the Turbo codeword are 262 and 268, respectively.

Numerical Results

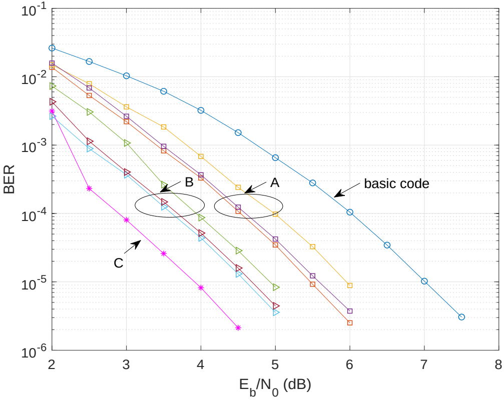


Figure: BER results of self-superposition convolutional codes and the corresponding convolutional code $\mathcal{C}[3, 2, 3]$ with codeword length 1542, where A, B and C represent three types of self-superposition convolutional codes with the generator matrices having 1, 2 and 3 interleavers, respectively.

Summary

- The proposed self-superposition transmission enhances the error performance of convolutional codes without changing the code rate and the codeword length;
- The proposed self-superposition convolutional codes can be efficiently encoded/decoded with the proposed algorithms;
- Self-superposition convolutional codes with randomly generated interleavers are suitable for short block length transmission.

Thank you!